

# The intrinsic degree of freedom for quasiparticle in thermodynamics with medium effects

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## Abstract

The thermodynamics with medium effects expressed by the temperature- and density-dependent effective mass of quasiparticle is studied. Series difficulties and many wrangles in references due to the extraordinary parameter dependence are addressed. A new independent intrinsic degree of freedom of quasiparticle  $m^*$  in the equation of reversible process is introduced to clear the ambiguity. We prove all results are self-consistent.

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It is generally accepted that the effective mass of particle will change with temperature and density due to its interaction with the environment, usually referred as the medium effects. This phenomenon is not only demonstrated by theoretical studies, for example, Brown-Rho scaling [1], Quantum Chromodynamics(QCD) sum rules [2], vacuum polarization Feynman diagrams calculated by Thermo-Field Dynamics [3, 4], *etc.*, but also by experiment [5].

Although the density- and temperature-dependent quasiparticle mass  $m^*(T, \rho)$  is commonly used to mimic the medium effects, difficulty emerges when we discuss the thermodynamics of the system with such quasiparticle. In thermodynamics, a proper choice of independent variables will have a suitable characteristic thermodynamical function, from which all the thermodynamic quantities can be obtained by partial derivatives without integration. For example, with variables temperature  $T$ , volume  $V$  and chemical potential  $\mu$ , the characteristic function is the thermodynamical potential  $\Omega = \Omega(T, V, \mu)$ . From the exact differential relation

$$d\Omega = -SdT - pdV - Nd\mu, \quad (1)$$

we have

$$S = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu}, \quad p = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu}, \quad N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,V}, \quad (2)$$

where  $S$ ,  $p$  and  $N$  are entropy, pressure and particle number respectively. Other thermodynamic quantities such as internal energy  $U$ , Helmholtz free energy  $F$ , enthalpy  $H$ , Gibbs function  $G$ , *etc.*, can also be calculated by the combination of the quantities we obtained, based on their definitions or relations.

But for the case with effective mass, the relativistic dispersion relation for a quasiparticle with energy  $\epsilon$  and momentum  $k$  becomes

$$\epsilon = (k^2 + m^*(T, \rho)^2)^{1/2}, \quad (3)$$

so  $\Omega$  is not only a function of  $T$ ,  $V$ ,  $\mu$ , but also depends explicitly on the quasiparticle mass,  $\Omega = \Omega(T, V, \mu, m^*(T, \rho))$ . How to tackle the thermodynamics self-consistently is still a problem. There have been many wrangles in present references [6, 7, 8, 9, 10, 11, 12]. The difficulty comes from the first and second laws of reversible process thermodynamics expressed in Eq.(1) and the partial derivatives as in Eq.(2). Obviously, some extra terms involving the derivatives of  $m^*$  will emerge. Unfortunately, these extra terms for different

treatments in different references contradict each other. For example, for ideal gas system of quasiparticle with effective mass  $m^* = m^*(\rho)$ , the pressure and energy density  $\varepsilon$  were given by

$$p = -\tilde{\Omega} \equiv -\frac{\Omega}{V}, \quad (4)$$

$$\varepsilon \equiv \frac{U}{V} = \tilde{\Omega} + \sum_i \mu_i \rho_i - T \frac{\partial \tilde{\Omega}}{\partial T}, \quad (5)$$

in Ref.[6]; given by

$$p = -\left(\frac{\partial(\tilde{\Omega}/\rho)}{\partial(1/\rho)}\right)_{T,\{\mu_i\}} = -\tilde{\Omega} + \rho\left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\}} \quad (6)$$

$$\varepsilon = \tilde{\Omega} - \rho\left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\}} + \sum_i \mu_i \rho_i - T\left(\frac{\partial \tilde{\Omega}}{\partial T}\right)_{\{\mu_i\},\rho}. \quad (7)$$

in Ref.[7]; and given by

$$p = -\tilde{\Omega} + \rho\left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\}}, \quad (8)$$

$$\varepsilon = \tilde{\Omega} + \sum_i \mu_i \rho_i - T\left(\frac{\partial \tilde{\Omega}}{\partial T}\right)_{\{\mu_i\},\rho}, \quad (9)$$

in Ref.[9]. For  $m^* = m^*(T, \rho)$ ,  $p$  and  $\varepsilon$  read

$$p = -\tilde{\Omega} - V \frac{\partial \tilde{\Omega}}{\partial V} + \rho \sum_i \frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial \rho}, \quad (10)$$

$$\varepsilon = \tilde{\Omega} - \sum_i \mu_i \frac{\partial \tilde{\Omega}}{\partial \mu_i} - T \frac{\partial \tilde{\Omega}}{\partial T} - T \sum_i \frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}, \quad (11)$$

in Ref.[11]. The ambiguity arises from the variable  $\rho$  in  $m^*$ , because it is not one of the characteristic variables in  $\Omega(T, V, \mu)$ . Even in Ref.[12], a supplement term  $\Omega_\alpha$  which satisfied  $\frac{\partial(\tilde{\Omega}+\Omega_\alpha)}{\partial \rho} = 0$  was introduced to compensate the density dependence of thermodynamical potential. The confusion in the market tells us this problem of common interest is still unsolved.

This letter involves from an attempt to clear above ambiguity and give a treatment to calculate the thermodynamic quantities from partial derivatives self-consistently.

Noticing that at a fixed instant of reversible process, the system is at an equilibrium state. Denote the temperature and density are  $T_0$  and  $\rho_0$  respectively, then the effective mass of the quasiparticle becomes constant  $m^*(T_0, \rho_0) = m_0$ . The system reduces to a usual ideal gas system with constant mass  $m_0$  quasiparticles. For this equilibrium state,

the corresponding thermodynamic quantities can be directly obtained. For example, for the case of one component Fermi system:

$$N \equiv \rho V = \sum_i g_i n_i = \sum_i \frac{g_i}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (12)$$

$$\Omega = - \sum_i g_i kT \ln(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)}), \quad (13)$$

$$U = \sum_i g_i n_i \epsilon_i = \sum_i \frac{g_i \sqrt{m_0^2 + k_i^2}}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (14)$$

$$G = N\mu = \sum_i g_i n_i \mu = \sum_i \frac{g_i \mu}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (15)$$

$$S = \frac{U - \Omega - G}{T}, \quad (16)$$

$$p = -\frac{\Omega}{V}, \quad (17)$$

where  $n_i$  is the particle number of the  $i$ th state and  $g_i$  is the corresponding degeneracy.

We see, from Eqs.(12)-(17), the contribution of medium effect is included in the effective value of mass, and appears in the exponential of the Fermi distribution. A remarkable property of these formulae is that the extra terms related to the partial derivative of  $m^*$  does not appear. This is reasonable because these thermodynamic quantities are functions of state, they do not depend on the change of the quasiparticle mass.

To compare above treatment with others given in Ref.[6, 7, 8, 9, 10, 11, 12], as an example, we calculate the entropy of the ideal quasiparticle system. Denote the entropy calculated by equilibrium state as  $S_{sta}$ . From Eq.(13)-(16) we have:

$$\begin{aligned} S_{sta} &= \frac{U - G - \Omega}{T} \\ &= \sum_i g_i \left[ \frac{n_i \sqrt{m_0^2 + k_i^2} - n_i \mu}{T} + k \ln(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)}) \right] \\ &= k \sum_i g_i \left[ n_i \beta (\sqrt{m_0^2 + k_i^2} - \mu) + \ln \left( \frac{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)}} \right) \right] \\ &= k \sum_i g_i \left[ n_i \ln \left( \frac{1}{n_i} - 1 \right) + \ln \left( \frac{1}{1 - n_i} \right) \right] \\ &= -k \sum_i g_i [n_i \ln(n_i) + (1 - n_i) \ln(1 - n_i)]. \end{aligned} \quad (18)$$

This is a familiar formula for equilibrium state whose physical meaning is transparent. While we denote the entropy calculated by the partial derivative of  $\Omega$  following Eqs.(1) and (2) as

$S_{der}$ , we have

$$\begin{aligned} S_{der} &= -\left(\frac{\partial\Omega(T, V, \mu, m^*(T, \rho))}{\partial T}\right)_{V, \mu} \\ &= -\left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial T}\right)_{V, \mu, m^*} - \left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial m^*}\right)_{T, V, \mu} \left(\frac{\partial m^*(T, \rho)}{\partial T}\right)_{V, \mu}, \end{aligned} \quad (19)$$

and for the same equilibrium state with  $m^*(T_0, \rho_0) = m_0$ ,

$$\begin{aligned} & -\left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial T}\right)_{V, \mu, m^*=m_0} \\ &= k \sum_i g_i \ln(1 + e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT}) + kT \sum_i g_i \frac{e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT} \left(\frac{\sqrt{m_0^2 + k_i^2} - \mu}{kT^2}\right)}{1 + e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT}} \\ &= k \sum_i g_i \ln\left(\frac{1}{1 - n_i}\right) + k \sum_i g_i n_i \ln\left(\frac{1}{n_i} - 1\right) \\ &= -k \sum_i g_i [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] = S_{sta}. \end{aligned} \quad (20)$$

The first term of Eq.(19) is just the result given by equilibrium state. The difference between these two treatments is significant. They can not be accorded together. Noticing that the contribution of medium effect at equilibrium state is included within the value of  $m^*$  in the distribution, and the entropy describing disorder of quasiparticles in a system dose not depend on the intrinsic quantity such as effective mass of quasiparticle, the correctness of  $S_{sta}$  is obvious.

Furthermore, if we calculate the particle number  $N$  from Eqs.(1) and (2), we obtain

$$N_{der} = N_{sta} - \left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial m^*}\right)_{T, V, \mu} \left(\frac{\partial m^*(T, \rho)}{\partial \mu}\right)_{T, V}, \quad (21)$$

where  $N_{sta}$  is calculated from the equilibrium state shown in Eq.(12). Since  $\rho = N/V$  is usually taken as a constraint to decide the value of  $\mu$ ,  $(\frac{\partial m^*(T, \rho)}{\partial \mu})_{T, V}$  dose not vanish if  $m^*$  depends on  $\rho$  explicitly, so the extra term in Eq.(21) modifies the particle number of the system. Since the number of quasiparticle will not be changed by the medium effect, the result of Eq.(21) is incorrect.

To get a self-consistent calculation between equilibrium state and reversible process in thermodynamics, we are compelled by above results to introduce an intrinsic degree of freedom  $m^*$  for quasiparticle in thermodynamic system with medium effect, and rewrite Eq.(1) as:

$$d\Omega = -SdT - pdV - Nd\mu + Xdm^*, \quad (22)$$

then Eq.(2) becomes

$$S = -(\frac{\partial \Omega}{\partial T})_{V,\mu,m^*}, \quad p = -(\frac{\partial \Omega}{\partial V})_{T,\mu,m^*}, \quad N = -(\frac{\partial \Omega}{\partial \mu})_{T,V,m^*}, \quad X = (\frac{\partial \Omega}{\partial m^*})_{T,V,\mu}, \quad (23)$$

where  $X$  is an extensive quantity corresponding to the intensive variable  $m^*$ . In Eq.(22), the intrinsic degree of freedom  $m^*$  for quasiparticle has been added as an independent variable in thermodynamic system. The quantities  $S$ ,  $p$  and  $N$  shown in Eq.(23) are in agreement with the results obtained by the formulae of equilibrium state in Eqs.(12)-(17).

Usually the thermodynamic parameters such as  $S$ ,  $p$ ,  $T$ ... depend on the whole system. They are independent with the intrinsic property of the particle or subsystem, no matter the subsystem is a simple point particle or a quasiparticle with inner structure and different intrinsic properties. Ordinary thermodynamic variables depend on the collection of the subsystem only. Similarly, the mass is an intrinsic quantity of a particle, it does not affect on collective thermodynamic properties of the whole system. When we consider the medium effect, and summarize this effect into the effective mass  $m^*(T, \rho)$  under quasiparticle approximation, the dynamic interaction can be concentrated on the effective mass  $m^*$  of quasiparticle by using the finite temperature quantum field calculation [13]. The thermodynamic variables cannot describe these micro dynamic interactions. We must choose new variables to represent these dynamic interactions or the medium effect. Obviously, the effective mass  $m^*$  appears as the suitable independent variable. As we know that the number of independent thermodynamic variables of a normal system obey the Gibbs' phase rule [14], here we extend the Gibbs' phase rule to include the medium effect by adding a new independent variable, the intrinsic degree of freedom for quasiparticle, effective mass  $m^*$ .

In summary, we have introduced a new intrinsic degree of freedom  $m^*$  for the system of quasiparticle. We have proved that the thermodynamic quantities calculated by the partial derivatives concerning this independent variable are in agreement with those obtained by the equilibrium state. The difficulties and controversies in previous references are removed. Since the effective mass of quasiparticle is quite widely used in many aspects of physics, we hope our new statement may help the study on the thermodynamic properties of such system.

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